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Halley's comet: its size and decay rate

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Summary. The outgassing rates inferred from the 1910 apparition and the brightness decay over the previous two millenia are compatible with the minimum nuclear brightness currently observed if the comet nucleus is small, 1.8–2.7 km radius with an albedo of 0.1–0.2. Outgassing is faster than from a bare nucleus of dirty H_2O-ice, which is attributed either to a hot microdust coma or to an organic polymer composition. Halley's comet will decay away within another 45–65 apparitions.

1 Introduction

The failure to spot Halley's comet with modern detectors and the largest telescopes in 1981, and its unexpected faintness on recovery in 1982, has led to a belief that the nucleus is very dark with visual albedo $A_V \simeq 0.05$ (e.g. Hughes 1985). This albedo is comparable to values for the Moon, the darkest C3 asteroids and carbonaceous chrondrites, surfaces that have had lengthy exposure to Solar System plasma and dust impacts. It is not obviously appropriate to comets composed of dirty-ice mixtures, which shed an outer layer several metres deep on each perihelion passage (e.g. Fernández & Jockers 1983). During comet Halley's mere 70 quiescent years between visits to the inner Solar System, pitting of the surface by interplanetary dust impacts affect much less than 1 per cent of its area, while photoprocessing and proton impacts (Strazulla, Calcagno & Foti 1984) are similarly incapable of changing the optical properties in the available time.

Hughes (1985) used the observed brightness and estimates of the outgassing rate to infer the nuclear size, presuming a low albedo and discounting his earlier order-of-magnitude for the comet's decay over two millenia from historical records (Hughes 1983) on the grounds that an unfashionably high albedo is implied. A higher subsequent estimate of the decay rate (Ferrin 1984) makes matters even worse. It therefore seems worth examining more closely whether a small nucleus solution with higher if unfashionable values of albedo is compatible with the available observations of brightness and with a rapid decay over a few tens of returns (Ferrin 1984). While favouring Ferrin's value derived on more sophisticated analysis giving a fading rate of $\Delta H = 0.055$ mag per apparition to within ±30 per cent, rather than the less precise $\Delta H = 0.0$ (Broughton 1979) or $\Delta H = 0.00$–0.04 (Hughes 1981), we observe that these earlier results are not incompatible with Ferrin's.

For estimating the size, Hughes (1985) calculated a mean brightness as appropriate to a rotating body, although much of the observed brightness fluctuation by up to a factor 5 appears non-periodic (West & Pedersen 1984; Sekanina 1984). The absolute magnitudes over 1982–84, as corrected by Sekanina for distance, phase and photometric band, range between 13.1 and a lower limit of 14.9. Supposing the brighter phases to be due to some coma activity (Sekanina 1984), and allowing for observational errors and possible rotation effects, the nucleus mean brightness would plausibly imply an absolute magnitude in the range $H_0=14.5–14.75$ (*cf.* 13.68 ± 0.06 from the unweighted mean calculated by Hughes). These values translate to an equivalent radius (spherical asteroid)

$$R=(0.74–0.83)\,A_V^{-0.5}\,\text{km}. \tag{1}$$

2 Outgassing rates

The reference model for comet Halley (Newburn 1981) based on a mainly H_2O-ice composition, has source strength of 1×10^{30} molecules s^{-1} at perihelion. Recent work has the comet twice as bright on the post-perihelion leg (Marcus 1983; Bortle & Morris 1984), so we take

$$Q_m=\alpha\cdot2\times10^{19}\,\text{molecules s}^{-1} \tag{2}$$

with an uncertainty factor $\alpha=½–2$. The solar radiation suffices to sublimate H_2O-ice at the rate $(1-A)\times1.7\times10^{18}$ molecules $cm^{-2}\,s^{-1}$ at 1 AU (Delsemme 1982), so an area πR^2 at $q=0.5871$ AU perhelion gives

$$Q=1.55\times10^{19}(1-A)R^2\phi\quad s^{-1} \tag{3}$$

with ϕ some efficiency factor, allowing for (small) fractions lost in radiation, conducted into the interior, and intercepted by the coma.

The photometric model of Newburn (1981) gives outgassing rates away from perihelion, but has the brightness dominated by the secondary product C_2, such that

$$Q[H_2O]\sim\sqrt{Q[C_2]}\sim r^{-1.65}.$$

As sublimation processes always give steeper variation than r^{-2}, this slow decrease with the heliocentric r is unphysical; so let us instead use $Q\sim r^{-n}$ with $n=2$ for simple H_2O-ice at small r and $n=3\pm0.5$ of recent comets observed in the UV (Festou 1982). The total outgassing around a complete orbit $r=q(1+e)/(1+e\cos\theta)$ is evaluated as

$$S=Q_m q^n\int d\theta/r^n\dot{\theta}=Q_m^y q^{3/2}(1+e)^{1.5-n}\langle(1+e\cos\theta)^{n-2}\rangle. \tag{4}$$

Here, the unit of perihelion distance q has been taken as AU, Q_m^y is the peak rate in units of molecules yr^{-1} and $\langle\ldots\rangle$ denotes the average over an orbit:

$$\frac{1}{\pi}\int_0^\pi(1+e\cos\theta)^{n-2}d\theta=1+(n-2)(n-3)e^2/2!2$$
$$+(n-2)(n-3)(n-4)(n-5)3e^4/4!8+\ldots. \tag{5}$$

The photometric curve from the 1910 apparition, taking the total gas production as proportional to visual brightness and normalizing to the Divine (1981) peak flux, is given (Bortle & Morris 1984) by a function discontinuous at perihelion, as might correspond to some less volatile crust blown off at peak temperatures (Fernández & Jockers 1983):

$$Q=1\times10^{30}\,\text{molecules s}^{-1},\qquad n=4.4 \text{ pre-perihelion}$$

$$Q_\mathrm{m}=2.2\times10^{30}\,\text{molecules s}^{-1},\qquad n=3.1 \text{ post-perihelion.} \tag{6}$$

Taking a mean molecular weight of 23 to allow for components heavier than OH/H_2O and adding 50 per cent by mass of dust grains, the values (6) inserted in (4) give 1.0 and 3.8×10^{30} g per half orbit, pre- and post-perihelion respectively. Marcus (1983) gave a 20 per cent lower Q_m with $n=2.9$ post-perihelion, resulting in slightly lower integral loss. The total mass loss per apparition is therefore taken as

$$\Delta M=4.8\alpha\times10^{14}\,\text{g} \tag{7}$$

with $\alpha=½-2$ covering the biggest uncertainty – that of normalization.

3 Decay rate

The loss of mass is related to the decay in visual magnitude, assuming a uniform spherical body with unchanging albedo (*cf.* Hughes & Daniels 1983), by

$$\Delta M/M_0=-\Delta H/0.724. \tag{8}$$

The data on first naked-eye observations, corrected for twilight and selected against lunar interference (Ferrin 1984), give the value of ΔH (preferred to earlier results for reasons given by Ferrin and in the caption to Fig. 1). Since M is seen to vary significantly through the 1700 years, note that $\Delta M\propto R^2$ (ignoring the small, non-systematic fluctuations in perihelion q – Yeomans & Kiang 1981) which implies

$$M=M_0(1-N/K)^3$$

at the Nth apparition (counting negatively in the past), M_0 being the present (1985) mass and K=const, which with (8) leads to

$$\Delta H=0.724\{(1-N/K)^3-1\}. \tag{9}$$

The data selected by Ferrin fit functions of the form (9) with $K=45$–65 as depicted in Fig. 1. This implies the comet will disappear after 45–65 further apparitions – more than Ferrin's 39 because of our non-linear fit of equation (9).

The current mass loss of $3M_0/K$ per apparition now leads with (7) to

$$M_0=1.6\alpha K\times10^{14}\,\text{g},\qquad R_0=(\alpha K/\varrho)^{1/3}\times0.34\,\text{km} \tag{10}$$

with density ϱ in g cm^{-3}. When this value is inserted into (1) and with Q in (3) equated to its maximum value (2), we have

$$A_V=4.8\text{–}6.1(\varrho/K\alpha)^{2/3},\qquad (1-A)\phi_\mathrm{m}=114\alpha^{1/3}(\varrho/K)^{2/3}. \tag{11}$$

Here the albedo A for solar energy absorption is adequately approximated by A_V and ϕ_m is the peak conversion efficiency per nuclear area and H_2O molecule latent heat. Replacing K by its range 45–65 gives

$$A=0.30\text{–}0.47\,\varrho^{2/3}/\alpha^{2/3},\qquad (1-A)\phi_\mathrm{m}=7.0\text{–}9.0\,\alpha^{1/3}\varrho^{2/3}. \tag{12}$$

These expressions are quite severely restrictive. For a density as high as 1 g cm^{-3}, corresponding to slightly porous ice with a fraction of mineral material, values $\alpha>½$ mean

$$A>0.47\text{–}0.75,\qquad \phi_\mathrm{m}>10. \tag{13}$$

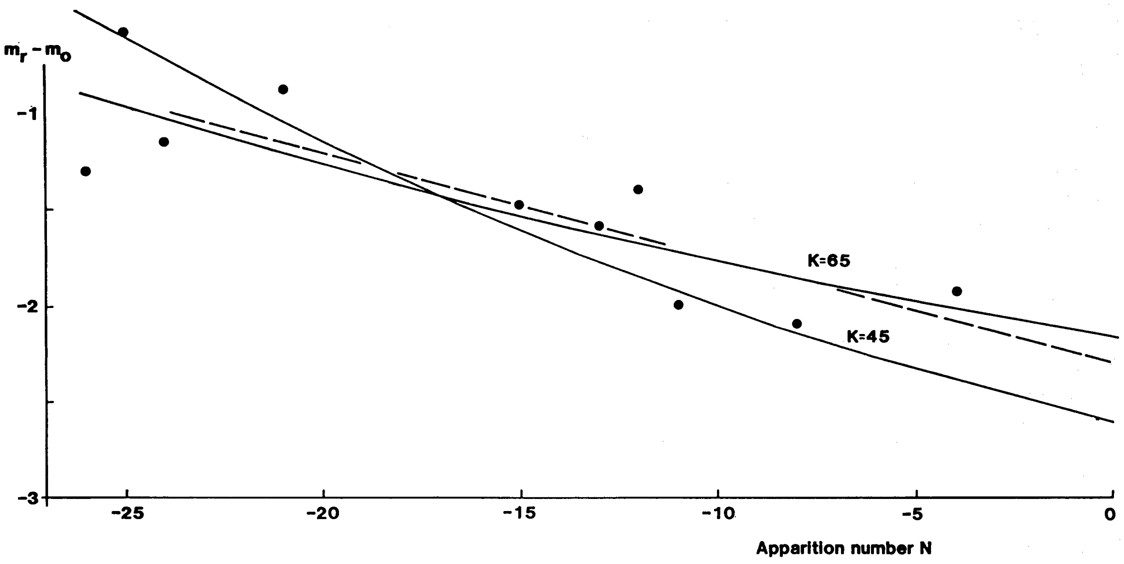

Figure 1. Brightness data selected and corrected by Ferrin (1984) fitted to curves of the form (9) to an arbitrary constant on the vertical scale. Note that Broughton (1979) selected a quite different subset of apparitions ($N=-26$, -20, -16, -13, -10, -8, -9) that includes only three of Ferrin's points, omitting instances close to the Sun rather than making twilight corrections, and found no systematic change in H_0. Hughes (1983) added the post-perihelion recoveries $N=-25$, -24, -21, -15, -12, -11 with an 0.8-mag correction, the pre-perihelion points $N=-6$, -5 with unfavourable Moon and $N=-4$ with unfavourable Sun. While his result for ΔH is formally consistent with zero systematic change, that is only marginally tenable being dependent on a single datum ($N=-5$) deviating by $2\frac{1}{2}\sigma$.

A low-density nucleus such as lightly compacted snow and/or organic material, $\varrho \simeq 0.3\,\mathrm{g\,cm^{-3}}$, (12) gives

$$A=0.21\text{–}0.33, \qquad \phi_m=3.2\text{–}4.8 \qquad \text{for } \alpha=\tfrac{1}{2},$$

or

$$A=0.08\text{–}0.13, \qquad \phi_m=4.3\text{–}5.9 \qquad \text{for } \alpha=2. \tag{14}$$

4 Discussion

The $\varrho \gtrsim 1\,\mathrm{g\,cm^{-3}}$ solution (13) with high albedo and efficiency is implausible, but comets may have low density corresponding to lightly compacted 'snow'; indeed values 0.3–0.5 $\mathrm{g\,cm^{-3}}$ seem to be implied by the larger non-gravitational accelerations (Wallis & Macpherson 1981). Taking the 0.3 $\mathrm{g\,cm^{-3}}$ as the lowest probable density for a cohesive nucleus, relations (12) imply that peak efficiencies ϕ_m exceed 3, as the values of ϕ_m in (13–14) exemplify. Values $\phi_m \gtrsim 3$ are in principle admissible. For instance, a coma of volatile grains would utilize the solar flux over a larger area. Alternatively, a coma of small refractory and absorptive grains of optical depth $\lesssim 1$ in the IR heats up an ice nucleus via IR re-radiation (Hellmich 1981; Weissman & Kieffer 1981) by up to 2.6 times (Weissman & Kieffer 1984). While a spherically symmetric coma as assumed in their models is implausible (both the formation and its stability are questionable) and a sunward conical coma would not enhance the heat flux by so much, this model does have the merit of giving a power index n around 3 (i.e. $Q \sim r^{-3}$) comparable to observed outgassing behaviour. However, their $\phi_m \lesssim 2.6$ is unattractive in implying values of α, ϱ or K a little outside our specified ranges for dirty snow.

The second possibility, compatible with the higher values of ϕ_m in (14), is a composition dominated by polymerized organics rather than H_2O-ice. Organic molecules accreted under interstellar conditions are likely to take on polymerized forms. As exemplified by polyoxymethylene polymers and copolymers (Vanýsek & Wickramasinghe 1975; Cooke & Wickramasinghe 1977), the effective sublimation temperature is high corresponding to end-capping of polymer chains by covalent bonds of order 1 eV. However, the detachment of a single terminal molecule in a chain would lead to unravelling of an entire chain which may comprise some 30 molecules. The effective attachment energy per molecule is then $\lesssim 0.05$ eV, 10 times lower than the sublimation energy per H_2O molecule from ice at 200 K. Also, the range of threshold energies corresponding to various end-capping bonds means that gas production increases more steeply on approach to the Sun than $Q \sim r^{-2}$ of H_2O-ice.

The range $K=45$–65 further apparitions before the comet's disappearance should not be taken as firm upper and lower limits, given the dispersion and corrections to the data of Fig. 1. Hughes' (1983) central value for the decay at 0.02 mag per apparition giving $K=140$ could be taken as an extreme estimate (see caption to Fig. 1). Hughes (1985) suggests that the constancy of non-gravitational forces perturbing comet Halley's orbit (Yeomans & Kiang 1981) implies a long lifetime; however, their effect measured by the surface to mass ratio would have changed by a scarcely detectable 28–37 per cent.

Typical comet lifetimes have been suggested that are much higher. Fernández (1984) gives 500–1000 apparitions as consistent with the deficit in comets with small q. Kresák (1981) found averages of 360 revolutions for all Jupiter-family comets, but 100 revolutions only for the subset with $q<1$ AU (excluding the exceptional comet Encke), based on three comets that have disappeared. For eight Halley-type comets (excluding Halley), Kresák gives a lifetime of 80 revolutions, based on 41 apparitions, 1 disappearance. The statistical uncertainty is large. Comet Halley itself, with a significantly smaller q, would outgas faster (as $q^{n-1.5}$ per revolution, so 2–3 times the average with $q=1.1$), but its diameter is presumably bigger by a similar factor.

Thus we conclude that our value of 55 ± 10 further revolutions is somewhat low, but nevertheless quite compatible with Kresák's figures. Because of the low density and high outgassing efficiency compared with ice, the comet loses several tens of metres of surface per revolution rather than several metres (Fernández & Jockers 1983).

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